AN EMPIRICAL STUDY OF MULTIVARIATE STRATIFICATION

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ABSTRACT

An empirical study comparing univariate, bivariate and trivariate stratification is presented for a multipurpose survey. Results indicated that substantial variance reductions can be produced by using multivariate rather than univariate stratification.

Key words: Multivariate stratification; Multivariate measure of efficacy; Multipurpose surveys

CONTENTS

SUMMARY

Analysis of the 1978 Farm Production Expenditure Survey (FPES) data showed that multivariate stratification can provide substantial gains in efficiency over univariate stratification for a multipurpose survey. In almost all instances, bivariate stratification was superior to univariate stratification. In all instances, trivariate stratification was more efficient than univariate stratification. Trivariate stratification was superior to bivariate stratification in most instances. Analysis also showed that for multivariate stratification more strata are needed than is generally considered sufficient for univariate stratification.

INTRODUCTION

As the List Sampling Frame System comes into existence for ESCS, the potential use of multipurpose surveys needs exploration. The List Sampling Frame provides ESCS with the capability of combining several single-purpose surveys into one survey to obtain information on a variety of agricultural characteristics such as crop acreages, livestock inventories and farm production expenditures. Combining single-purpose surveys into a multipurpose survey would reduce respondent contacts, decrease survey costs and lighten office workload. One example of a multipurpose survey is the Probability Crop and Livestock Survey being initiated in the North Carolina SSO to provide county estimates for crop acreages and livestock inventories.

In order to obtain reliable estimates for many of the survey items in a multipurpose survey, the technique of multivariate stratification needs investigation. Traditionally, stratification is done with one variable that is related to a few of the survey items. However, for a multipurpose survey it is unlikely that stratification by a single variable would provide reliable estimates for the variety of agricultural items being estimated. For example, stratification by land in farm may be beneficial for estimating acreages for major crops, but is not as useful for estimating livestock inventories. Therefore, a stratification design based on more than one variable may be more appropriate for a multipurpose survey.

Several approaches have been suggested for combining several variables available for stratification. These include cluster analysis, principal components analysis and cross-classification of variables. . Cluster analysis is not a realistic approach to stratifying the List Sampling Frame because clustering algorithms are much too expensive on large data sets. Earlier work by Kish and Anderson [3] with trivariate normal distributions and with three empirical studies showed that cross-classification of stratifiers usually produced greater variance reductions than principal components analysis. Therefore, the cross-classification approach to multivariate stratification was selected as the approach to be analyzed in this report.

The purpose of this report was to investigate the benefits of multivariate stratification in a survey providing information on crop acreages, livestock inventories and farm production expenditures. The studies by Kish and Anderson [3] mentioned previously and research by Thomsen [4] with normal, rectangular and exponential distributions have shown that the benefits from using multivariate stratification rather than univariate stratification were generally nontrivial.

DATA SOURCE

Data from the 1978 FPES conducted by ESCS was used to evaluate the usefulness of multivariate stratification. FPES data was chosen because it provided information on a variety of agricultural items (crop acreages, livestock inventories and farm production expenditures). Five states were included in the study. They were Illinois, Iowa, Minnesota, Missouri and Wisconsin. Farm operations with 100 to 500 total harvested acres or 50 to 500 hogs or 50 to 500 cattle were used in the analysis. For this data set, the five-state average number of total harvested acres, hogs and cattle was 289 acres, 127 hogs and 82 cattle.

The number of farm operations in the data set was 616. Fourteen of these operations did not have any harvested acreage, 354 had no hogs and 186 did not have any cattle. Of the 616 operations, 427 were resident farm operators from 225 segments from the economic area frame. The remaining 189 operations were selected from a list of large livestock operators maintained in each of the five SSO's and from the 1975 Social Security list of agricultural employees.

SURVEY ITEMS

Fourteen survey items from the 1978 FPES were included in the analysis. These items were selected so that data on three categories (crop acreages, livestock inventories and farm production expenditures) was available for analysis. The survey items for each of these categories along with the abbreviation for the survey items and categories that will be used at times in this report were:

CROP ACREAGES (ACREAGES):

Corn harvested for grain (CORN)

Small grains harvested for grain (SMGRAIN)

Soybeans harvested for beans (SBEANS)

Hay harvested (HAY)

Total harvested acres (HARVACRE): includes 'all' crops harvested during 1978.

LIVESTOCK INVENTORIES (LIVESTOCK):

Inventory of cattle and calves (CATTLE)

Cattle and calves on feed for slaughter (COF)

Inventory of hogs and pigs (HOGS)

FARM PRODUCTION EXPENDITURES (EXPENSES):

Seed and plant expenses (SPEX)

Fertilizer, lime and soil conditioner expenses (FLCEX)

Agricultural chemical expenses (CHEMEX)

Expenses for livestock and poultry purchased (LPEX)

Feed expenditures (FEEDEX)

Wages and contract labor costs (WAGES)

STRATIFICATION

Three variables were chosen as stratification variables. They were total harvested acreage, hog inventory and cattle inventory. Harvested acreage was selected because it was felt that this variable would not only be beneficial for reducing the variances of crop acreages that were correlated with harvested acreage but also for lowering the variances of farm production expenditures related to harvested acreage such as fertilizer expenses. Land in farm and cropland acreage were not considered in lieu of harvested acreage because neither variable was available from the 1978 FPES. Hog and cattle inventories were each chosen as stratifiers because of the importance of reducing the variances of livestock items. In addition, these livestock variables may be useful in estimating more reliably farm production expenditures related to cattle or hog inventory such as feed expenses.

To test the usefulness of stratifying with more than one variable, the variances of the fourteen survey items were computed when stratified by harvested acreage, hogs and cattle individually and compared with the variances generated from the three bivariate stratifiers (harvested acreage and hogs, harvested acreage and cattle, hogs and cattle) and the trivariate stratifier (harvested acreage and cattle and hogs) using various numbers of strata.

It must be pointed out that for each of the three stratification variables the 1978 FPES data was used to construct the strata boundaries rather than control data from a previous FPES or other sources. That is, the 1978 FPES data for total harvested acreage, hog and cattle inventories was used not only to construct the strata but also as three of the fourteen survey items to be analyzed. Therefore, the variances of survey items will be lower than the variances would have been if stratification was done using historic data. However, this fact should not affect the comparisons between multivariate and univariate stratification in this report.

The cum $\sqrt{f}$ rule was used to construct the strata boundaries for total harvested acreage, hogs and cattle. Cochran has shown that this rule works well for theoretical and actual distributions [1]. Tortora, Rockwell and Ciancio [5] have shown that the cum $\sqrt{f}$ rule performs as well as or better than other stratification rules when stratifying the ESCS area frame. For multivariate designs, the cum $\sqrt{f}$ rule was used separately for each stratification variable involved as has been done in research by Kish and Anderson [3] and Thomsen [4]. For example, if four hog by cattle strata were desired (two hog strata by two cattle strata), the cum $\sqrt{f}$ rule was used to generate two hog strata (0 to 195 hogs and more than 195 hogs) and two cattle strata (0 to 105 cattle and more than 105 cattle). The four hog by cattle strata (2x2) would then be:

(1) 0-195 hogs and 0-105 cattle

(2) 0-195 hogs and > 105 cattle

(3) > 195 hogs and 0-105 cattle

(4) > 195 hogs and > 105 cattle

As many as 20 strata were constructed. This may appear to be a large number of strata since the literature shows that relatively small gains in variance reduction are generally produced for more than six to eight strata unless the correlation between the survey item and stratification variable is very high, which is rarely the case [2]. It must be kept in mind that this rule of thumb pertains only to univariate stratification. When stratifying with two or more variables, variance reductions might not be small when using more than six to eight strata. A maximum of 20 strata was decided upon for comparisons between multivariate and univariate stratification designs in order to reduce computer expenses.

The numbers of strata used for each of the univariate stratifiers were 4,6,8,9,10,12,14,15,16,18 and 20. These 11 strata numbers were selected because they conveniently allowed for analytic comparisons with the following designs for each of the three bivariate stratifiers: 2x2, 2x3, 3x2, 2x4, 4x2, 3x3, 2x5, 5x2, 2x6, 3x4, 4x3, 6x2, 2x7, 7x2, 3x5, 5x3, 2x8, 4x4, 8x2, 2x9, 3x6, 6x3, 9x2, 2x10, 4x5, 5x4 and 10x2 and the following designs for the trivariate stratifier: 2x2x2, 2x2x3, 2x3x2, 3x2x2, 2x2x4, 2x4x2, 4x2x2, 2x3x3, 3x2x3, 3x3x2, 2x2x5, 2x5x2 and 5x2x2.

UNIVARIATE ANALYSIS

Shown in Table 1 is the correlation coefficient between each of the three stratification variables and the fourteen survey items. For each survey item, the largest correlation coefficient is boxed off. In three instances the correlation coefficient was 1.000 since the stratification variable was also the survey item, as mentioned earlier.

Table 1: Correlation Coefficient Between Each Stratification Variable and Survey Item.1/

Survey Item	Stratification Variable: Harvested Acreage	Cattle	Hogs
ACREAGES:			
HARVACRE	1.000	.255	.171
CORN	.776	.181	.263
SMGRAIN	.358	.106	-.017 NS
SBEANS	.729	-.063 NS	.102
HAY	.213	.362	-.074 NS
LIVESTOCK:			
CATTLE	.255	1.000	.026 NS
COF	.251	.663	.083
HOGS	.171	.026 NS	1.000
EXPENSES:			
SPEX	.574	.195	.125
FLCEX	.727	.318	.263
CHEMEX	.624	.154	.210
LPEX	.303	.641	.140
FEEDEX	.115	.346	.461
WAGES	.250	.176	.112

1/ NS denotes that the correlation coefficient was not significantly different from zero.

Inspection of the variance of each of the fourteen survey items when stratified by either harvested acres, hogs or cattle for the 11 distinct numbers of strata yielded the following three results: (1) The variances for total harvested acreage, acres of corn, small grains and soybeans, seed and plant costs, fertilizer, lime and soil conditioner expenses, chemical costs and expenses for wages and contract labor were smallest when the univariate stratifier was harvested acres. (2) With cattle as the stratification variable, the variances for cattle inventory, cattle on feed, hay acreage and expenditures for livestock and poultry purchased were smallest.

(3) Variances for hog inventory and feed expenses were smallest when hog inventory was the univariate stratifier. Therefore, as expected, no one stratification variable provided the smallest variance for all of the fourteen survey items. In Table 2 the variance of each survey item is given for each stratification variable when there were four strata. The magnitude of the variance of each survey item for each stratifier can be seen from this table. For each survey item the smallest variance is boxed off. Notice that the stratifier generating the smallest variance for each survey item in Table 2 also has the largest correlation coefficient with that survey item in Table 1.

Table 2: The Variance of Each Survey Item for Each Stratifier When There Were Four Strata.

Survey Item	Stratification Variable		
	Harvested Acreage	Cattle	Hogs
ACREAGES:			
HARVACRE	17,739	[illegible],259	61,165
CORN	9,662	36,882	16,493
SMGRAIN	3,422	[illegible]	3,765
SBEANS	10,86[illegible]	[illegible],832	14,808
HAY	3,993	[illegible],150	4,157
LIVESTOCK:			
CATTLE	17,153	6,328	18,426
COF	8,699	6,127	9,069
HOGS	77,764	81,691	20,657
EXPENSES:			
SPEX	12,399,581	14,865,191	15,453,084
FLCEX	33,590,852	46,647,620	49,908,889
CHEMEX	7,453,283	9,441,527	9,435,497
LPEX	2,[illegible],308,258	2,207,580,870	2,838,839,529
FEEDEX	1,175,515,329	1,111,094,846	1,055,564,231
WAGES	163,106,107	168,0[illegible],229	173,850,840

If one of the three stratifiers reliably estimated all the important survey items and imprecise estimates could be tolerated for the remaining survey items, a single stratifier would suffice. Unfortunately, this is not the case when dealing with survey items such as soybean acreage, cattle and hog inventories, which are all very important survey items but each cannot be reliably estimated with only one stratifier due to the small correlations between at least one of these survey items and any single stratification variable. (See Table 1). Therefore, for a multipurpose survey it may be more desirable to stratify with several variables in order to satisfactorily estimate the important survey items. The merits of multivariate stratification will be examined in the following sections.

MULTIVARIATE MEASURE OF EFFICACY

To compare multivariate and univariate stratification, a multivariate measure of efficacy was computed. This measure compares the variances of the survey items between the univariate and multivariate stratifiers when the same number of strata is used. For example, a multivariate measure of efficacy was computed to compare the variances of the fourteen survey items when eight strata were created with cattle as the univariate stratifier and when four cattle strata crossed with two hog strata (4x2) were created as the bivariate stratifier. No comparison between multivariate and univariate stratifiers was done when the number of strata was not the same. The form of the multivariate measure of efficacy is [3]:

$$E_p = \frac{\sum_g I_g V_{pg}/V_{og}}{\sum_g I_g V_{1g}/V_{og}} \quad \text{where}$$

E_p = multivariate measure of efficacy for p stratification variables

V_{pg} = variance of the g^{th} survey item using p stratifiers

V_{1g} = variance of the g^{th} survey item using one stratifier

V_{og} = variance of the g^{th} survey item if no stratification was used

I_g = relative importance or importance index of the g^{th} survey item

where $\sum_g I_g = 1$

A multivariate measure of efficacy, E_p, equal to one means that there is no advantage to using multivariate stratification. If E_p is greater than one, the univariate stratifier is preferred. Finally, if E_p is less than one, multivariate stratification is more efficient.

Since there were fourteen survey items, an importance index, I_g, had to be assigned to each survey item in order to use the multivariate measure of efficacy. If there are n survey items and each is considered of equal importance to the data users, then $I_g = 1/n$. If the n survey items are not of equal importance to the data users, e.g. if hog inventory is more important than feed expenses, then unequal indices would be much more appealing. Both unequal and equal indices were examined. The approach taken to assign unequal indices to the survey items will now be discussed.

The unequal importance indices were assigned in a two-step process. First, weights adding to one were assigned to the survey items in each of the three categories (crop acreages, livestock inventories and farm production expenditures) and the three categories were assigned weights adding to one.

Data from the 1977 Farm Income Statistics Bulletin published in September of 1978 was used to assign weights to the survey items in each category. The five-state total values of corn, soybeans, small grains and hay were used to determine each crop's importance. Total harvested acreage was subjectively assigned a weight equal to the most important crop in the five states because it was felt that assigning a weight to harvested acreage based on the five-state total crop value would be reflecting too much importance for harvested acreage. The five-state total value was also used to determine the weight for hogs, cattle and cattle on feed. Finally, the five-state total farm production expenditures for each of the six expenditure survey items were used to assign weights to the expenditures. The fourteen weights were rounded to the nearest five percent whenever possible. The weights for the survey items within each of the three categories were:

ACREAGES		LIVESTOCK		EXPENSES	
HARVACRE:	.30	HOGS :	.50	FEEDEX:	.35
CORN :	.30	CATTLE:	.40	FLCEX :	.20
SBEANS :	.30	COF :	.10	FPEX :	.20
SMGRAIN :	.08			SPEX :	.10
HAY :	.02			WAGES :	.10
				CHEMEX:	.05

Next, each of the three categories was assigned a weight. In order to examine the sensitivity of the multivariate measure of efficacy when several reasonable weights were applied to the three categories, the following four methods of weighting the three categories were analyzed:

Method 1:	ACREAGES :	1/3	Each category was considered to be of equal importance to the data users.
	LIVESTOCK:	1/3	
	EXPENSES :	1/3	
Method 2:	ACREAGES :	.40	Acreage and livestock data were of equal importance and each was twice as important to the data users than expenditure information.
	LIVESTOCK:	.40	
	EXPENSES :	.20	
Method 3:	ACREAGES :	.45	Acreage estimates were of most value to the users, then livestock estimates, then expenditure estimates.
	LIVESTOCK:	.35	
	EXPENSES :	.20	
Method 4:	ACREAGES :	.35	The data users considered livestock estimates to be most important, then acreages, then expenditures.
	LIVESTOCK:	.45	
	EXPENSES :	.20	

Thus, for methods 1 through 4 the importance index, I_g, for the g^{th} survey item was the weight assigned to the survey item within its category multiplied by the weight assigned to the category.

Finally, a fifth method was analyzed that gave equal weight to each of the survey items. Since there were fourteen survey items, each index, I_g, was 1/14. Method 5 did not utilize the data from the 1977 Farm Income Statistics Bulletin as did Methods 1 through 4. Method 5 was the least desirable of the methods described because in reality the importance of the survey items generally is not the same to the data users. The fifth method was included for completeness. Since there were five acreage survey items, three livestock survey items and six expenditure survey items, method 5, in effect, assigned a weight of 5/14 to crop acreages, 3/14 to livestock inventories and 6/14 to farm production expenditures.

Notice that in none of the five methods was one category given most of the weight, e.g. a weight of .9 to acreages. The reason this was not done was that it was assumed in this study that most of the importance was not limited to one category such as crop acreages. For if this was the case (an index of .9 to acreages), it would be of little value, if any, to also stratify by a variable such as hog or cattle inventory that is poorly correlated with acreage items.

BIVARIATE ANALYSIS

The analysis comparing the various bivariate and univariate stratifiers is summarized in Tables A-1 through A-6 in the APPENDIX. Presented in the tables is the bivariate measure of efficiacy, E_2, for each of the five methods of weighting the survey items for each of the stratification schemes. For example, the number .949 in Table A-1 for method 4 and design 3x2 refers to the bivariate measure of efficacy between the univariate design stratified by harvested acres with six strata and the bivariate design stratified by three strata of harvested acres and two strata of cattle.

Inspection of Tables A-1 through A-6 shows that E_2 was always less than one for methods 1, 2 and 4. E_2 was less than one in 158 of 162 instances for method 3. In 141 of 162 instances, E_2 was less than one for method 5, which as mentioned earlier was the least appealing weighting method. Therefore, bivariate stratification was almost always more efficient than univariate stratification.

The greatest gains in variance reductions occurred when there were 20 strata. For methods 1 through 4 the following bivariate designs were the most efficient compared to the univariate designs: 5 harvested acres x 4 cattle strata, 5 harvested acres x 4 hog strata and 5 hog x 4 cattle strata. When these designs were compared to the corresponding univariate stratifiers with twenty strata the bivariate measure of efficacy averaged about .82. This represents an overall reduction in the variances and standard deviations for the fourteen survey items of about 18 and 10 percent, respectively.

For method 5 neither the 5x4 nor 10x2 design was consistently superior when there were 20 strata. Comparison of each best bivariate design to the univariate design with 20 strata yielded an average bivariate measure of efficacy of about .87 or an overall reduction in variances and standard deviations of about 13 and 7 percent, respectively.

Bivariate measures of efficacy were only computed for as many as 20 strata. If variance reductions are negligible for more than 20 strata when using a single stratifier and if variance reductions are not negligible when using two stratifiers, greater gains in variance reductions than have been shown with 20 or less strata would be realized if more than 20 strata were used. This hypothesis will now be examined.

It was mentioned earlier that for univariate stratification gains in efficiency were generally small for more than six to eight strata, but that this rule of thumb did not necessarily apply to multivariate stratification. In Table 3 the validity of this statement is checked by comparing the overall reduction in variances when going from 8 to 20 strata. Given in this table for the univariate and bivariate stratifiers is the quantity, $1 - S_{20}/S_8$,

where $S_i = \sum_{g=1}^{14} I_g V_{pg}/V_{og}$

and i = the number of strata.

When two stratifiers were involved, the most efficient designs with 8 and 20 strata were used to compute this quantity.

Table 3: The Quantity $1 - S_{20}/S_8$, for the Univariate and Bivariate Stratifiers for Each Method of Weighting.

Stratification Variable(s)	Method 1	Method 2	Method 3	Method 4	Method 5
HARVACRE	.034	.035	[illegible]	.033	.033
CATTLE	.034	.036	.038	.040	.026
HOGS	.046	.050	.047	.054	.023
HARVACRE x CATTLE	.119	.131	[illegible]	.131	.106
HARVACRE x HOGS	.096	.114	[illegible]	.115	.055
HOGS x CATTLE	.093	.105	.092	.118	.050

Referring to Table 3, variance reductions averaged less than four percent going from 8 to 20 strata when stratification was done with one variable, but averaged more than 10 percent when stratification involved two variables. This finding supports the statement made earlier that for multivariate stratification more than six to eight strata may be necessary.

Further analysis of each univariate stratifier showed that variance reductions were nill when more than 20 strata were used. If variance reductions are substantial using bivariate stratification with more than 20 strata, gains in efficiency from bivariate stratification will be substantial for more than 20 strata since variance reductions for univariate stratification were nill for more than 20 strata.

In order to reduce computer expenses, it was decided to select one of the three bivariate stratifiers for further analysis. The selection criterion was to choose the bivariate stratifier that generally had the smallest value for the quantity, $\sum_g I_g V_{2g}/V_{og}$, for the best design when there were 20 strata. This quantity is shown for each method in Table 4. The stratifiers, harvested acres and hogs, generally had the smallest values for this quantity. Only for method 5, the least appealing weighting method, was another set of stratifiers clearly superior. Therefore, the bivariate stratifiers, harvested acres and hogs, were selected.

Table 4: The Quantity, $\sum_g I_g V_{2g}/V_{og}$, for Each of the Three Bivariate Stratifiers for the Best Design When There Were 20 Strata

Stratification Variables	Strata	Design	Method 1	Method 2	Method 3	Method 4	Method 5
HARVACRE x CATTLE	20	5x4	.629	.599	.587	.611	.661
HARVACRE x HOGS	20	5x4	.623	.585	.579	.592	.709
HOGS x CATTLE	20	5x4	.650	.621	.650	.591	.742

Inspection of Tables A-2 and A-5 in the APPENDIX illustrates that it was better to have more strata for harvested acreage than hogs, but not necessarily many more, e.g. 6x3 performed better than 9x2 and 5x4 performed better than 10x2. Therefore, to determine if more than 20 strata might be useful for bivariate stratification the quantity, $\sum_g I_g V_{2g}/V_{og}$, was generated for strata numbers between 20 and 100, inclusive, when the number of strata for harvested acres was greater than or equal to the number of strata for hogs, but not more than three times greater. The maximum number of strata for harvested acreage was set at 10. A limit of 100 strata was imposed for analysis because there were only 616 observations in the data set.

The value of the quantity, $\sum_g I_g V_{2g}/V_{og}$, for each design and method of weighting is shown in Table A-7 in the APPENDIX. Notice that this quantity has not approached an asymptotic value for any of the methods even when as many as 100 strata were analyzed. It should be kept in mind that the stratification was based on 1978 FPES data rather than control data from a previous FPES or other sources. Therefore, the correlations of each of the two variables with the survey items were higher than they would be if the control data for stratification purposes came from historic data. This fact may be causing the quantity in Table A-7 not to stabilize as quickly as it would had historic data been used for stratification. A determination of how many strata would be sufficient for bivariate stratification was not done using this data set. This was not done because in reality control data for stratification comes from previous surveys and/or other sources such as criteria letters rather than the current survey as was done in this analysis. Therefore, the resulting number of strata needed for bivariate stratification would not have been realistic. Table A-7 was generated to illustrate that for bivariate stratification it may be justifiable to use much more than 20 strata since reductions in the variances may still be substantial.

The approximate asymptotic value of $\sum_g I_g V_{1g}/V_{og}$ was determined when harvested acreage was the univariate stratifier and then when hogs was the univariate stratifier. These asymptotic values were then used to compute the bivariate measure of efficacy when there were 20 and 100 strata to see if the overall reduction in variances was substantial going from 20 to 100 strata. For the univariate stratifiers, Table 5 shows the overall reduction in variances using 20 and 100 strata for each weighting method when the bivariate stratifiers, harvested acres and hogs, were used rather than each of the univariate stratifiers. For 20 strata, the best design was used to compute the overall reduction in variances. The reductions are shown as a percentage in the table.

Table 5: Overall Reduction in Variances When Bivariate Stratifiers Used Rather than Univariate Stratifiers for 20 and 100 Strata.

Stratifier	Strata	Method 1	Method 2	Method 3	Method 4	Method 5
		%	%	%	%	%
HOGS	20	20.2	22.7	25.5	19.7	20.7
	100	36.5	40.2	42.4	38.0	32.0
HARVACRE	20	13.2	16.0	13.5	17.8	5.0
	100	31.1	34.9	33.0	36.9	18.3

For methods 1 through 4, the average overall reductions in variances using bivariate stratification rather than univariate stratification with hogs for 20 and 100 strata were about 22 and 39 percent, respectively. For method 5, the overall reductions were about 21 and 32 percent for 20 and 100 strata, respectively. When bivariate stratification was used rather than harvested acres as the univariate stratifier the average overall reductions for 20 and 100 strata for methods 1 through 4 were about 15 and 34 percent, respectively.

For method 5, the overall reduction was 5 percent for 20 strata and about 18 percent for 100 strata. These findings illustrate that substantial gains in variance reduction for bivariate stratification can be made as the number of strata is increased well beyond 20. These findings are not intended to imply that 100 strata should be used for bivariate stratification. The comparison of variance reductions between 20 and 100 strata was done strictly for analytical reasons.

TRIVARIATE ANALYSIS

The trivariate measure of efficacy, E_3, for each design and weighting method is shown in Tables A-8, A-9 and A-10 in the APPENDIX. In every instance, the trivariate measure of efficacy was less than one. This means that stratification based on the three variables was always more efficient than stratification based on any single variable.

The best trivariate design with 20 strata was 5 harvested acres x 2 hog x 2 cattle strata. For methods 1 through 4 the average E_3 was about .76. This represented an overall reduction in variances and standard deviations of about 24 and 13 percent, respectively. For method 5 the average E_3 was higher at about .83. Clearly, the gains using trivariate rather than univariate stratification were nontrivial.

Comparison of the best bivariate and trivariate efficacy measures when there were 20 strata demonstrated that for methods 1 through 4 stratification with three rather than two variables was always more efficient. For method 5, three variables were superior to two variables four out of six times. Thus, for this data set it was better to use three variables rather than two for stratification.

Examination of trivariate stratification as the number of strata increased well beyond 20 was not done due to the limited number of observations in the data set. Intuitively, since variance reductions were substantial for bivariate stratification when the number of strata was increased well beyond 20, variance reductions should also be substantial for trivariate stratification as the number of strata surpasses 20. Finally, since variance reductions should become negligible for bivariate stratification before trivariate stratification as the number of strata is increased, the benefits of trivariate stratification over univariate or bivariate stratification should be greater as the number of strata becomes very large. Therefore, much greater gains from trivariate stratification may be realized than were stated in this report.

CONCLUSIONS AND RECOMMENDATIONS

Results indicated that substantial variance reductions can be produced in a multipurpose survey by using multivariate rather than univariate stratification. Bivariate stratification was almost always more efficient than univariate stratification, and trivariate stratification was always superior to univariate stratification. In most instances, trivariate stratification was more efficient than bivariate stratification. Results also showed that for multivariate stratification more strata are needed than is generally considered adequate for univariate stratification.

Since the information used for stratification purposes in this study was not historic data as is the case on the List Sampling Frame, it is recommended that the merits of multivariate stratification in a multipurpose survey be evaluated using historic stratification data. In January, 1980 a survey is being conducted by the North Carolina SSO to provide county estimates for crop acreages and livestock inventories. Since historic data is being used for stratification purposes in this survey and since it is a multipurpose survey it is recommended that multivariate stratification be investigated using North Carolina's survey data. In addition to examining the cross-classification approach to multivariate stratification that was used in this study, principal components analysis should also be investigated as an approach to multivariate stratification.

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APPENDIX

Tables A-1 Through A-10

Table A-1

Bivariate measure of efficacy when the univariate stratifier is harvested acreage and the bivariate stratifiers are harvested acreage and cattle for the 5 methods of weighting.

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
4	2x2	.989	.988	1.005	.971	.992
6	2x3	.981	.979	1.006	.953	.990
	3x2	.970	.964	.980	.949	.980
8	2x4	.966	.964	.996	.934	.978
	4x2	.960	.952	.965	.939	.971
9	3x3	.958	.950	.972	.929	.974
10	2x5	.968	.968	1.004	.934	.977
	5x2	.945	.935	.947	.924	.959
12	2x6	.960	.959	.997	.923	.972
	3x4	.936	.927	.953	.902	.953
	4x3	.936	.924	.943	.906	.954
	6x2	.946	.935	.945	.925	.959
14	2x7	.960	.963	1.004	.925	.968
	7x2	.944	.933	.943	.923	.957
15	3x5	.928	.920	.951	.893	.943
	5x3	.919	.908	.924	.892	.936
16	2x8	.954	.956	.999	.916	.963
	4x4	.907	.894	.917	.874	.923
	8x2	.932	.921	.931	.912	.946
18	2x9	.952	.954	.998	.914	.958
	3x6	.915	.911	.942	.882	.926
	6x3	.909	.896	.911	.883	.926
	9x2	.922	.911	.920	.902	.935
20	2x10	.941	.940	.984	.899	.956
	4x5	.886	.873	.897	.850	.910
	5x4	.875	.859	.876	.844	.898
	10x2	.912	.900	.909	.891	.930

Table A-2

Bivariate measure of efficacy when the univariate stratifier is harvested acreage and the bivariate stratifiers are harvested acreage and hogs for the 5 methods of weighting.

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
4	2x2	.960	.951	.974	.929	1.016
6	2x3	.954	.940	.971	.910	1.029
	3x2	.936	.922	.944	.901	.997
8	2x4	.952	.937	.973	.903	1.039
	4x2	.927	.912	.932	.894	.985
9	3x3	.921	.900	.927	.875	1.003
10	2x5	.952	.936	.975	.899	1.046
	5x2	.914	.897	.914	.880	.977
12	2x6	.951	.933	.975	.894	1.052
	3x4	.915	.892	.923	.863	1.006
	4x3	.911	.888	.914	.864	.986
	6x2	.910	.892	.909	.876	.974
14	2x7	.950	.933	[illegible]	.892	1.054
	7x2	908	.890	[illegible]	.874	.970
15	3x5	.913	.891	[illegible]	.858	1.011
	5x3	.896	.872	[illegible]	.850	.974
16	2x8	.952	.936	.982	.893	1.059
	4x4	.901	.879	.908	.851	.985
	8x2	.901	.883	.901	.868	.962
18	2x9	.940	.923	.969	.881	1.049
	3x6	.905	.881	.916	.848	1.007
	6x3	.879	.855	.877	.836	.956
	9x2	.896	.879	.895	.864	.957
20	2x10	.933	.915	.962	.872	1.052
	4x5	.887	.862	.893	.834	.985
	5x4	.868	.840	.865	.822	.963
	10x2	.881	.862	.878	.848	.950

Table A-3

Bivariate measure of efficacy when the univariate stratifier is <u>cattle</u> **and the bivariate stratifiers are** <u>cattle and harvested acreage</u> **for the 5 methods of weighting.**

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
4	2x2	.926	.917	.898	.938	.934
6	2x3	.905	.900	.862	.918	.920
	3x2	.916	.903	.885	.922	.928
8	2x4	.891	.873	.840	.906	.907
	4x2	.897	.884	.867	.901	.914
9	3x3	.883	.863	.837	.890	.906
10	2x5	.875	.855	.820	.892	.894
	5x2	.897	.885	.868	.902	.911
12	2x6	.874	.852	.814	.892	.891
	3x4	.865	.842	.812	.874	.887
	4x3	.866	.845	.821	.870	.886
	6x2	.887	.874	.859	.890	.904
14	2x7	.866	.843	.803	.884	.883
	7x2	.881	.870	.855	.886	.893
15	3x5	.841	.818	.784	.854	.859
	5x3	.849	.830	.807	.854	.865
16	2x8	.859	.836	.796	.878	.874
	4x4	.836	.812	.783	.841	.853
	8x2	.879	.867	.853	.882	.890
18	2x9	.851	.828	.787	.871	.867
	3x6	.840	.815	.779	.853	.858
	6x3	.845	.828	.805	.852	.858
	9x2	.880	.868	.853	.883	.888
20	2x10	.847	.824	.783	.867	.863
	4x5	.813	.787	.754	.821	.833
	5x4	.823	.799	.773	.827	.844
	10x2	.874	.861	.848	.875	.887

Table A-4

Bivariate measure of efficacy when the univariate statifier is cattle **and the bivariate stratifiers are** cattle and hogs **for the 5 methods of weighting.**

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
4	2x2	.942	.929	.898	.919	.997
6	2x3	.936	.916	.927	.904	1.009
	3x2	.918	.901	.912	.889	.979
8	2x4	.929	.908	.920	.895	1.012
	4x2	.896	.878	.891	.865	.958
9	3x3	.903	.879	.893	.865	.987
10	2x5	.927	.905	.918	.891	1.019
	5x2	.896	.879	.893	.865	.958
12	2x6	.921	.897	.911	.883	1.017
	3x4	.898	.873	.889	.857	.987
	4x3	.878	.854	.870	.837	.958
	6x2	.885	.867	.881	.852	.949
14	2x7	.916	.892	.906	.876	1.015
	7x2	.876	.859	.874	.843	.939
15	3x5	.883	.858	.874	.841	.976
	5x3	.868	.845	.863	.827	.942
16	2x8	.912	.889	.904	.873	1.010
	4x4	.870	.845	.863	.827	.955
	8x2	.865	.849	.865	.832	.925
18	2x9	.912	.888	.903	.873	1.011
	3x6	.885	.858	.875	.841	.983
	6x3	.857	.833	.851	.814	.935
	9x2	.869	.853	.868	.837	.925
20	2x10	.903	.879	.895	.863	1.009
	4x5	.841	.816	.837	.795	.934
	5x4	.861	.836	.857	.815	.947
	10x2	.858	.842	.859	.823	.919

Table A-5

Bivariate measure of efficacy when the univariate stratifier is <u>hogs</u> and the bivariate stratifiers are <u>hogs and harvested acreage</u> for the 5 methods of weighting.

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
4	2x2	.900	.894	.875	.915	.892
6	2x3	.868	.858	.832	.885	.858
	3x2	.885	.875	.856	.894	.885
8	2x4	.841	.829	.800	.859	.830
	4x2	.864	.851	.835	.868	.875
9	3x3	.845	.827	.803	.852	.847
10	2x5	.838	.824	.790	.859	.821
	5x2	.872	.860	.843	.877	.880
12	2x6	.828	.813	.778	.850	.812
	3x4	.828	.810	.782	.839	.822
	4x3	.832	.813	.790	.837	.839
	6x2	.865	.851	.835	.868	.877
14	2x7	.823	.807	.772	.845	.805
	7x2	.861	.846	.831	.862	.874
15	3x5	.814	.792	.763	.824	.811
	5x3	.830	.810	.788	.832	.842
16	2x8	.817	.800	.764	.839	.798
	4x4	.817	.796	.770	.823	.817
	8x2	.864	.848	.833	.864	.879
18	2x9	.815	.799	.761	.838	.796
	3x6	.800	.777	.746	.811	.796
	6x3	.823	.800	.779	.823	.838
	9x2	.855	.839	.824	.854	.873
20	2x10	.810	.793	.756	.833	.793
	4x5	.798	.773	.745	.803	.803
	5x4	.816	.793	.769	.819	.822
	10x2	.858	.842	.828	.857	.878

Table A-6

Bivariate measure of efficacy when the univariate stratifier is hogs and the bivariate stratifiers are hogs and cattle for the 5 methods of weighting.

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
4	2x2	.943	.940	.944	.937	.929
6	2x3	.913	.909	.915	.903	.898
	3x2	.931	.924	.930	.918	.926
8	2x4	.876	.870	.878	.861	.864
	4x2	.908	.899	.908	.891	.913
9	3x3	.899	.889	.897	.880	.895
10	2x5	.887	.883	.892	.874	.863
	5x2	.917	.909	.918	.900	.919
12	2x6	.871	.867	.876	.858	.851
	3x4	.864	.853	.865	.842	.859
	4x3	.883	.873	.884	.862	.885
	6x2	.907	.897	.906	.888	.912
14	2x7	.865	.862	.872	.851	.845
	7x2	.905	.895	.905	.885	.913
15	3x5	.862	.852	.865	.838	.854
	5x3	.877	.865	.876	.852	.885
16	2x8	.851	.847	.859	.834	.830
	4x4	.856	.844	.857	.830	.857
	8x2	.897	.888	.898	.876	.907
18	2x9	.856	.852	.864	.840	.831
	3x6	.844	.833	.847	.817	.839
	6x3	.871	.858	.871	.844	.883
	9x2	.897	.888	.898	.877	.908
20	2x10	.849	.846	.859	.832	.827
	4x5	.852	.841	.857	.824	.852
	5x4	.832	.820	.837	.803	.840
	10x2	.894	.884	.895	.872	.907

Table A-7

$\sum_g I_g V_{2g}/V_{og}$ for harvested acres x hogs for each method of weighting.

Strata	Design	Method 1	Method 2	Method 3	Method 4	Method 5
20	5x4	.623	.585	.579	.592	.709
21	7x3	.618	.580	.571	.590	.694
24	6x4	.611	.572	.564	.580	.696
	8x3	.616	.579	.568	.589	.691
25	5x5	.616	.578	.573	.582	.709
28	7x4	.600	.559	.551	.568	.685
30	6x5	.604	.566	.559	.572	.694
32	8x4	.598	.557	.548	.567	.681
35	7x5	.586	.547	.539	.554	.673
36	6x6	.587	.545	.537	.552	.679
	9x4	.584	.542	.531	.552	.668
40	8x5	.584	.544	.536	.552	.670
	10x4	.574	.532	.522	.543	.662
42	7x6	.567	.523	.514	.531	.661
45	9x5	.570	.530	.521	.539	.657
48	8x6	.563	.519	.510	.528	.655
49	7x7	.575	.533	.527	.539	.667
50	10x5	.557	.517	.509	.526	.649
54	9x6	.561	.516	.507	.526	.652
56	8x7	.571	.530	.523	.537	.662
60	10x6	.546	.501	.491	.511	.641
63	9x7	.560	.517	.509	.525	.651
64	8x8	.554	.512	.504	.519	.651
70	10x7	.547	.504	.496	.513	.641
72	9x8	.553	.508	.500	.516	.648
80	10x8	.538	.494	.485	.502	.638
81	9x9	.549	.503	.494	.511	.648
90	10x9	.520	.477	.469	.484	.625
100	10x10	.495	.453	.448	.457	.602

Table A-8

Trivariate measure of efficacy when the univariate stratifier is harvested acreage and the trivariate stratifiers are harvested acreage, cattle and hogs for the 5 methods of weighting.

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
8	2x2x2	.910	.894	.933	.857	.973
12	2x2x3	.886	.861	.906	.819	.973
	2x3x2	.886	.867	.913	.825	.959
	3x2x2	.879	.856	.890	.824	.953
16	2x2x4	.869	.843	.891	.798	.966
	2x4x2	.858	.839	.890	.792	.934
	4x2x2	.856	.831	.862	.802	.928
18	2x3x3	.856	.830	.880	.783	.948
	3x2x3	.845	.814	.853	.779	.940
	3x3x2	.838	.814	.854	.778	.918
20	2x2x5	.852	.824	.873	.778	.960
	2x5x2	.845	.826	.878	.778	.922
	5x2x2	.821	.791	.819	.766	.904

Table A-9

Trivariate measure of efficacy when the univariate stratifier is cattle and the trivariate stratifiers are harvested acreage, cattle and hogs for the 5 methods of weighting.

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
8	2x2x2	.845	.820	.812	.828	.910
12	2x2x3	.819	.785	.780	.789	.904
	2x3x2	.819	.791	.786	.795	.891
	3x2x2	.812	.780	.766	.795	.886
16	2x2x4	.801	.765	.761	,769	.893
	2x4x2	.791	.762	.761	.763	.863
	4x2x2	.789	.754	.736	.772	.858
18	2x3x3	.791	.754	.752	.757	.879
	3x2x3	.781	.741	.729	.753	.871
	3x3x2	.774	.740	.730	.751	.851
20	2x2x5	.791	.754	.752	.756	.890
	2x5x2	.785	.756	.756	.756	.855
	5x2x2	.763	.724	.705	.745	.839

Table A-10

Trivariate measure of efficacy when the univariate stratifier is hogs and the trivariate stratifiers are harvested acreage, cattle and hogs for the 5 methods of weighting.

STRATA	DESIGN	METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5
8	2x2x2	.826	.812	.801	.824	.820
12	2x2x3	.806	.785	.776	.794	.811
	2x3x2	.807	.791	.781	.800	.799
	3x2x2	.800	.780	.762	.800	.795
16	2x2x4	.788	.763	.756	.772	.802
	2x4x2	.778	.760	.755	.765	.775
	4x2x2	.776	.752	.731	.775	.770
18	2x3x3	.779	.754	.748	.760	.789
	3x2x3	.769	.740	.725	.756	.782
	3x3x2	.762	.740	.726	.754	.764
20	2x2x5	.783	.758	.752	.764	.801
	2x5x2	.777	.760	.757	.764	.770
	5x2x2	.755	.728	.705	.752	.755